Preemption and Delay: Debt Financing with Roll Over Risk

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November 20, 2017

Abstract

Short-term debt that is exposed to roll-over risk creates pre-emption motives among investors. I show that these preemption motives can facilitate the efficient liquidation of underperforming investment projects when investors are heterogeneously informed. The ability to observe other investors' funding decisions can inefficiently delay the withdrawal of funding - waiting has an option value as it reveals new information. Providing financial rewards for early withdrawers incentivises the revelation of private information and counteracts the option value of waiting. The optimal debt maturity structure depends on the quality of investors' private information. First-best liquidation decisions can be implemented via the correct mix of short- and long-term debt.

1 Introduction

When investing under uncertainty investment decisions undertaken by other investors carry double weight. Firstly, they may directly impact the returns of the common investment project. Failure to roll over short term debt by one party, for example, can inflict considerable costs on the remaining investors if it triggers costly liquidation. Investment decisions with several parties involved will generally involve such payoff externalities. Secondly, when investment decisions are based on private information, observing other investors' actions reveals additional information about the investment's payoff. Such informed actions thus create an informational externality by reducing the payoff uncertainty of those who are able to observe them.

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The strategic implications of these two externalities for dynamic interactions between investors can be quite distinct. Informational externalities are well-known to cause strategic delays (e.g. Chamley and Gale (1994)). Waiting has an option value as it allows to gather additional information by observing other investors' actions. This option value reduces the incentives to act immediately upon receiving private information. Investors try to outwait each other. Payoff externalities, on the other hand, can create preemption motives when late movers' payoffs are negatively affected by previously taken actions. Banks runs (e.g. Diamond and Dybvig (1983)) are a prominent consequence of such negative payoff externalities.

We study the interplay of such informational and payoff externalities in a model where several parties invest in a common project with risky returns. We show that the preemption motive created by negative payoff externalities can counteract the incentives to delay actions that arise from the ability to observe other investors. This counteracting force is shown to facilitate information revelation and thereby improve the efficiency of investment decisions.

In our model two investors hold claims, consisting of a mix of short term and long term debt, to the future risky payoffs of an investment project and can receive private information about these payoffs. Short term debt has to be rolled over at intermediate stages of the project and failure to do so by either investor triggers liquidation with a fixed liquidation value. Thus the decision to liquidate by one investor impacts the payoff of the other investor. This is the source of the payoff externality in our model. Short term debt which has not been rolled over is senior to both long term debt and rolled-over short term debt in the case of liquidation. The ratio of short term to long term debt thus controls the size of the payoff externality. The ability to observe the roll over decisions of the other investor allows inference about his information and thereby creates informational externalities. We solve for the unique perfect Bayesian equilibrium in symmetric strategies of this game and show that an appropriately chosen mix of short term and long term debt can guarantee efficient liquidation decisions. We show that the optimal level of the payoff externalities created through short term debt crucially depends on the liquidation value of the project and the quality of the private information received by investors.

Related Literature The topic of observational learning has triggered a large literature. Important early contributions are Chamley and Gale (1994), Gul and Lundholm

(1995) and Bikhchandani et al. (1998). A key insight of these models is that the ability to observe others' behaviour generates an option value from waiting and can thereby impedes the revelation of privately held information. Gale (1996) provides an overview. Our model is closely related to Weeds (2002) who analyses a model of investment under uncertainty where preemption motives interact with incentives to delay caused by option values. However, in her work the option value of waiting does not arise from the ability to observe other players' actions but from a commonly observed stochastic process that reveals information about the profitability of investment over time. Frisell (2003) also models a setting where these two forces are at work, analysing a product placement game. However his focus is on understanding the timing of moves rather than on the issue of information revelation. Gu (2011) analyses a model of fundamental bank runs in the tradition of Allen and Gale (2002). In his model depositors receive both information about their consumption preferences and the payoff of an illiquid asset their bank is invested in. Depositors then have to decide whether to withdraw or keep their savings in the bank. Gu's setup includes both payoff externalities arising through the possibility of bank runs as well as informational externalities coming from the ability to observe other depositors' withdrawal decision. However his agents are atomistic and only a measure zero set of them has private information. Additionally informed agents do not move simultaneously which weakens preemption motives. Our setup with a finite number of actors and simultaneous moves allows for richer strategic interactions and consequently yields equilibrium behaviour that differs from Gu's analysis. We will see that in our model mixed strategies play an important role, whereas Gu's equilibrium is in pure strategies. Furthermore our focus is on the optimal financial structure of the investment in the presence of payoff and informational externalities, a topic Gu does not deal with.
Technically, the equilibrium construction in this paper is closely related to the approach used in Murto and Välimäki (2011) which looks at information aggregation in an exit game when privately informed players can observe each other exit decisions. However in their model exit decisions do not cause any payoff externalities, which facilitates the equilibrium analysis considerably.

The rest of the paper is organised as follows. Section 2 introduces the setup of our investment game, sections 3 solves for the unique perfect Bayesian equilibrium of the game. Section 4 derives the efficient investment policy and section 5 shows how it can be implemented through an appropriate mix of short term and long term debt claims. Section 6 discusses assumptions of the model. Section 7 concludes.

2 Model Setup

Project Characteristics We consider an investment project with uncertain payoff. The project can either be successful and pay out Y units at maturity, or it is unsuccessful in which case its payoff at maturity is zero. Whether the project is successful or not depends on the underlying state of the world $\theta \in \{0, 1\}$ where state 1 ($\theta = 1$) implies success and state 0 ($\theta = 0$) failure. The prior probability of success is $0 < \mu_0 < 1$.
Time evolves in discrete periods $t = 0, 1, 2, ...$ At $t = 0$ nature chooses the state of the world θ with $\mathbb{P}(\{\theta = 1\}) = \mu_0$. At each point in time $t > 0$, if still active, the investment project matures with probability $0 < \gamma < 1$ and pays out in the successful state only. If the project does not mature in period t it can be liquidated. Liquidation yields $0 < L < Y$ units irrespective of the state of the world θ. If the project has neither matured nor been liquidated in period t it carries on into the next period $t + 1$.

Investors Investors are risk neutral and have a discount rate of zero. Each investor's claims consist of a mix of short term and long term debt with face value d^s and d^l respectively. Long term debt entitles its owner to a payment of d^l when the project matures. Short term debt can be claimed in every period t with a promised payment of d^s. Alternatively an investor can decide to roll over his short term debt in which case he is entitled to a payment of d^s in the next period $t + 1$. Roll over decisions for short term debt are made simultaneously by all active investors.
The number of active investors depends on the state of the world θ. In state 0 there are two active investors, each of whom has to make a roll over decision for his short term debt in every period. In state 1 there is at most one investor with a roll over decision to make. Given that the project is unsuccessful in state 0, the presence of two active investors then indicates failure of the project. Hence having an active roll over decision to make is a negative signal for an individual investor. His beliefs concerning state 1 are given by $\mu < \mu_0$. A given investor does not know whether there is a second investor present who has a roll over decision to make. [1]

[1] To generate such beliefs one could, for example, assume that there are two potentially active investors and that in state 0 both of them are active. In state 1 however at most one of them is chosen to be active. An investor is chosen to be active with probability $0 < q \leq 1$ and each investor has an equal probability to be active in state 1. An individual investor can only observe that he has an active decision to make, but does not know whether the same is true for the second investor. By Bayes' Rule we would have

$$\mu = \frac{(1/2)q\mu_0}{(1/2)q\mu_0 + (1 - \mu_0)} < \mu_0$$

The lower q, the more informative is the fact that one has to make a roll over decision. We can think of q as the probability of any investor receiving a negative signal about the outlook of the project, when in fact the project is successful.

	exit	roll over
exit	l, l	$\phi, 2l - \phi$
roll over	$2l - \phi, \phi$	

Table 1 Payoff matrix in case of liquidation

Liquidation of project If at least one investor refuses to roll over his short term debt, the project has to be liquidated with liquidation revenue L. Claimed short term debt is senior to both long term debt and rolled over short term debt. The latter two are of equal seniority. Hence in case of a unilateral withdrawal by one investor, the investor who refuses to roll over receives

$$\min\{d^s, L\} + \max\left\{0, \frac{L - d^s}{2}\right\}$$

whereas the investor who has rolled over his short term debt receives

$$\max\left\{0, \frac{L - d^s}{2}\right\}.$$

If both investors refuse to roll over their short term debt in a given period, both receive half of the liquidation revenue, that is $L/2$.

In what follows we assume that a unilaterally withdrawing investor will receive the full face value of his short term debt, that is $d^s \leq L$. Furthermore to simplify notation we define

$$\phi = d^s + \frac{L - d^s}{2} \tag{1}$$

which stands for the amount received by a unilaterally withdrawing investor. Also define as $l = L/2$ the per-capita liquidation revenue. Payments in case at least one investor refuses to roll over his short term debt are then given by the payoff matrix in Table 1.

If the project matures in period t, an investor is paid $d^s + d^l$ in state 1 and zero in state 0. In the following we assume that the final payoff of the project in case of success is $Y = 2$ and each active investor in state 1 receives half of this, that is $d^s + d^l = 1$.

3 The Liquidation Game

An individual investor does not know whether he faces an active co-investor or not. He can however draw inference about the state of the world from the absence of liquidation up to the current period. If active investors do not roll over their short term debt with probability 1, the absence of liquidation in previous periods constitutes good news. It increases the likelihood of state 1 and thus the probability that the investment project has a positive payoff at maturity. However to the extent that observing the survival of the project provides valuable information on its eventual success and discount rates are zero, an individual investor has an incentive to wait in order to gather additional information. Waiting might dominate the withdrawal of short term financing and in equilibrium nothing can be learnt about the state of the world by observing the survival of the project. Investors would ignore their private information when making investment decisions. We show that if the cost of being preempted is sufficiently high and active investors' information is sufficiently precise, that is they are sufficiently pessimistic about the success of the project, such a situation cannot occur in equilibrium. Investors will always be able to learn about the state of the project from the absence of its liquidation in previous periods.

We now analyse the dynamic game played by active investors, starting in period $t = 1$ and lasting until either the investment project matures or at least one investor withdraws his short term funding. The equilibrium concept will be *perfect Bayesian equilibrium* (PBE) and we will restrict our attention to symmetric strategies. An investor's information set at time t only consists of whether the project is still active or not. As the withdrawal of short term funding by either investor terminates the game, an individual investor's strategy $\boldsymbol{\sigma}$ then simply specifies an exit probability $\sigma_t \in [0, 1]$ for any time period t in which the game is still active. Given an equilibrium strategy $\boldsymbol{\sigma}$ beliefs $\boldsymbol{\mu}$ are then calculated using Bayes' Rule

$$\mu_{t+1} = \frac{\mu_t}{1 - \sigma_t(1 - \mu_t)} \tag{2}$$

with $\mu_1 = \mu$. The pair $(\boldsymbol{\sigma}, \boldsymbol{\mu})$ is a PBE of the game if for any time t in which the game is still active, σ_t is a best response to $\{\sigma_j\}_{j \geq t}$ given beliefs μ_t and beliefs are updated using the equilibrium strategy according to (2).

We will now show that this game has a unique perfect Bayesian equilibrium in symmetric strategies. Whether liquidation occurs in equilibrium depends on investors' initial beliefs μ. For sufficiently optimistic beliefs investors will always roll over their short term debt and the project will be carried out until maturity. In particular if the payoff from unilaterally withdrawing short term financing, ϕ, is lower than the expected eventual payoff of the project, μ, rolling over until maturity is preferred to withdrawing if the other investor does so, too. For sufficiently pessimistic beliefs the only equilibrium outcome is immediate refusal to roll over. Here, even if the opponent's behaviour is maximally informative, that is he liquidates with probability 1 and thus the state of the world could be learnt by waiting one more period, an active investor prefers to withdraw short term funding. Waiting would yield $\mu + (1-\mu)(2l-\phi)$ whereas withdrawing would result in a payoff of $\mu\phi + (1-\mu)l$. Equating these two payoff determines the critical value $\hat{\mu}$ for beliefs below which immediate withdrawal is the only equilibrium. For intermediate values of beliefs μ every active investor rolls over short term funding with strictly positive probability less than 1. Proposition 1 characterises the unique perfect Bayesian equilibrium in symmetric strategies, which is then proven to be an equilibrium of this game in a sequence of lemmata.

Proposition 1. *The liquidation game has a unique perfect Bayesian equilibrium in symmetric strategies. There exists a $\hat{\mu} \in (0, \phi)$ such that an informed player refuses to roll over short term debt if $\mu < \hat{\mu}$, refuses to do so with probability $0 < \sigma_1 < 1$ in the first period and never thereafter if $\mu \in (\hat{\mu}, \phi)$ and continues to finance the project until maturity with probability 1 if $\mu > \phi$ where*

$$\hat{\mu} = \frac{\phi - l}{1 - l} \quad \textit{and} \quad \sigma_1 = \frac{\phi - \mu}{(1-\mu)l}$$

$V(\mu_t; \boldsymbol{\sigma})$ will designate the equilibrium value of the subgame starting at t for an active player given that the equilibrium strategy is $\boldsymbol{\sigma}$ and current beliefs of the player are μ_t. We have

$$\begin{aligned} V(\mu_t; \boldsymbol{\sigma}) = \max \{&[1-(1-\mu_t)\sigma_t]\phi + (1-\mu_t)\sigma_t l, \\ &(1-\mu_t)\sigma_t(2l-\phi) + [1-(1-\mu_t)\sigma_t][(1-\gamma)\mu_{t+1} + \gamma V(\mu_{t+1}; \boldsymbol{\sigma})]\} \end{aligned}$$

where the first term is the expected payoff of liquidation in t and the second term is the expected payoff of continuation in t given the equilibrium strategy $\boldsymbol{\sigma}$.

We start by showing that if the expected payoff at maturity of the investment is sufficiently high, it will never be liquidated in equilibrium.

Lemma 1. *If $\mu_t > \phi$ no liquidation occurs in the subgame starting in period t.*

Proof. Suppose there exists a period $\tau \geq t$ such that $\sigma_\tau > 0$. Then the player has to weakly prefer liquidation to continuation in period τ, that is

$$\begin{aligned}[1-(1-\mu_\tau)\sigma_\tau]\phi + (1-\mu_\tau)\sigma_\tau l \\ \geq (1-\mu_\tau)\sigma_\tau(2l-\phi) + [1-(1-\mu_\tau)\sigma_\tau][(1-\gamma)\mu_{\tau+1} + \gamma V(\mu_{\tau+1};\boldsymbol{\sigma})]\end{aligned}$$

where $V(\mu_{\tau+1};\boldsymbol{\sigma})$ is the equilibrium payoff of the game in period $\tau+1$. But as $V(\mu_{\tau+1};\boldsymbol{\sigma}) \geq \mu_{\tau+1}$ [2] this requires that

$$\phi \geq (1-\mu_\tau)\sigma_\tau l + \mu_\tau$$

which cannot hold as $\mu_\tau > \phi$ was assumed. □

Next we show that for sufficiently pessimistic beliefs liquidation has to occur with strictly positive probability.

Lemma 2. *If $\mu_t < \phi$ then in equilibrium we have $\sigma_t > 0$.*

Proof. We consider two possibilities in turn: firstly continuation forever from t onwards and, secondly, continuation with probability 1 up to some period $k > t$ and liquidation with strictly positive probability thereafter.
To see that the first scenario cannot be an equilibrium, note that the equilibrium payoff of continuing forever would be μ_t whereas a deviation to exit with probability 1 in period t would yield $\phi > \mu_t$.
Next suppose in equilibrium continuation is played with probability 1 from period t until $k > t$ and $k+1$ is the first period with strictly positive exit probability. For $\sigma_{k+1} > 0$ we need $V(\mu_{k+1};\boldsymbol{\sigma}) = [1-(1-\mu_{k+1})\sigma_{k+1}]\phi + (1-\mu_{k+1})\sigma_{k+1}l$, which is the expected payoff from exiting in period $k+1$. As $\sigma_{t+j} = 0$ for all $0 \leq j \leq k-t$ we have $\mu_{k+1} = \mu_k = \mu_t$ by (2). Nothing is learned from observing continuation of the project. Thus in k continuation is a best response to $\sigma_k = 0$ if

$$\phi \leq (1-\gamma)\mu_t + \gamma\left\{[1-(1-\mu_t)\sigma_{k+1}]\phi + (1-\mu_t)\sigma_{k+1}l\right\}$$

But this inequality cannot hold if $\mu_t < \phi$. It follows from the two above observations that in any equilibrium we need $\sigma_t > 0$ as long as $\mu_t < \phi$. □

The next lemma establishes that there cannot be two consecutive periods such that a player mixes between exiting and rolling over in one period, and then exits with strictly positive probability in the following period.

[2] Playing "always continue" from $\tau+1$ onwards has an expected payoff of at least $\mu_{\tau+1} + (1-\mu_{\tau+1})(2l-\phi)$.

Lemma 3. *If* $\mu_t < \phi$ *it cannot be the case that* $0 < \sigma_t < 1$ *and* $\sigma_{t+1} > 0$.

Proof. In this case the player has to be indifferent between exiting and rolling over in period t. We thus need

$$V(\mu_t; \boldsymbol{\sigma}) = [1 - (1 - \mu_t)\sigma_t]\phi + (1 - \mu_t)\sigma_t l$$

Furthermore we have

$$V(\mu_t; \boldsymbol{\sigma}) = (1 - \mu_t)\sigma_t(2l - \phi) + [1 - (1 - \mu_t)\sigma_t][(1 - \gamma)\mu_{t+1} + \gamma V(\mu_{t+1}; \boldsymbol{\sigma})]$$

Combining these two conditions yields

$$(1 - \mu_t)\sigma_t(\phi - l) = [1 - (1 - \mu_t)\sigma_t][(1 - \gamma)(\mu_{t+1} - \phi) + \gamma (V(\mu_{t+1}; \boldsymbol{\sigma}) - \phi)]$$

the left hand side of which is strictly positive. However for $\sigma_{t+1} > 0$ we need $\mu_{t+1} \leq \phi$ or else continuation with probability 1 would be the only possible equilibrium of the subgame starting in period $t + 1$. For the equality to hold it must thus be the case that $V(\mu_{t+1}; \boldsymbol{\sigma}) > \phi$. But this contradicts the fact that liquidation with positive probability requires $V(\mu_{t+1}; \boldsymbol{\sigma})$ to equal the payoff from liquidation in period $t + 1$, which is less than ϕ. □

For intermediate values of beliefs the project cannot be liquidated with probability 1. Here the option value of waiting and learning the true state in the next period if the opposing player exits exceeds the benefits of early exit.

Lemma 4. *The project is liquidated with probability 1 in period* t *if and only if* $\mu_t + (1 - \mu_t)l < \phi$.

Proof. We start with the *only if* part of the statement. Suppose $\sigma_t = 1$. If no exit occurs in period t the player knows that $\theta = 1$ and will continue in all subsequent periods given that $\phi < 1$. Rolling over in period t thus yields $(1 - \mu_t)(2l - \phi) + \mu_t$. Immediate exit yields $\mu_t\phi + (1 - \mu_t)l$. It follows that rolling over in period t is strictly preferred to exit as long as $\phi < \mu_t + (1 - \mu_t)l$.

We now prove the *if* part of the statement. By Lemma 2 we know that in equilibrium we need $\sigma_t > 0$ and by Lemma 3 we also know that this implies $\sigma_{t+j} = 0$ for all $j > 0$. Thus for any $\sigma_t > 0$ exiting is strictly preferred to rolling over if

$$[1 - (1 - \mu_t)]\phi + (1 - \mu_t)\sigma_t l > (1 - \mu_t)\sigma_t(2l - \phi) + \mu_t$$

where the left hand side is the expected payoff from exiting in period t and the right hand side is the expected payoff from continuation in t. It follows that exiting is strictly preferred to rolling over if $\phi > (1 - \mu_t)\sigma_t l + \mu_t$. But this is true for any $\sigma_t > 0$ and consequently in equilibrium we necessarily have $\sigma_t = 1$. □

Define $\hat{\mu}$ as the belief such that $\hat{\mu} + (1 - \hat{\mu})l = \phi$. The above lemmata establish that if the initial belief μ is strictly lower than $\hat{\mu}$, then the only equilibrium in symmetric strategies is immediate exit with probability 1 in period 1. If $\mu > \phi$, then in equilibrium the project is never liquidated. For intermediate levels of beliefs ($\hat{\mu} < \mu < \phi$) the project has to be liquidated with strictly positive probability $0 < \sigma_1 < 1$ in period 1 in any equilibrium in symmetric strategies. Conditional on not having been liquidated then, it is never liquidated thereafter.

Lastly we derive the equilibrium exit probability σ_1 for $\hat{\mu} < \mu < \phi$. Continuation with probability 1 from period 2 onwards implies that $V(\mu_2; \sigma) = \mu_2$. For the player to be indifferent between exiting and rolling over in period 1 given this continuation value we need

$$\phi = (1 - \mu)\sigma_1 l + \mu \quad \Rightarrow \sigma_1 = \frac{\phi - \mu}{(1 - \mu)l}$$

As $0 < \phi - \mu < (1 - \mu)l$ we have $0 < \sigma_1 < 1$ as required. It remains to check that $\mu_2 \geq \phi$ or else continuation from period 2 onwards would not be an equilibrium. But by (2) and the assumption that $\phi \geq l$ we have

$$\mu_2 = \frac{\mu l}{\mu - (\phi - l)} \geq \phi$$

This concludes the proof of Proposition 1. □

4 Efficient Liquidation

As a benchmark for efficient liquidation of the project we consider the roll over policy a planner would choose for an active investor, not knowing whether another active investor is present or not. For each period t the roll over decision can only be based on the number of periods the project has been active without being liquidated. We thus restrict the planner to using the same information available to an individual active investor when making roll-over decisions. We also restrict the planner to symmetric roll over policies, that is he cannot specify policies that vary with the identity of the investor. These two restrictions on the planner's choice of policy appear natural if we want to find a benchmark against which to judge the economic efficiency of the previously derived symmetric PBE.

Given the above restrictions a roll over policy is then a vector $\{\lambda_t\}_{t=1}^{\infty}$ where $\lambda_t \in [0,1]$ specifies the probability with which an active investor refuses to roll over short term debt in period t given that the investment project has not been liquidated previously. The planner's problem can be formulated as a dynamic programming problem where the investor's belief μ_t constitutes the state variable in period t which evolves according to

$$\mu_{t+1} = \frac{\mu_t}{1-\lambda_t(1-\mu_t)} \tag{3}$$

using Bayes' Rule and the fact that an active co-investor would follow the suggested roll over policy. A given λ_t then implies an expected liquidation probability of $\mu_t\lambda_t + (1-\mu_t)[1-(1-\lambda_t)^2]$ in period t. Either there is only one active investor (μ_t) in which case liquidation occurs with probability λ_t or the other investor has a roll over choice as well ($1-\mu_t$) and the project is liquidated if at least one of the investors exits. This happens with probability $1-(1-\lambda_t)^2$. In case of continuation the project pays out with probability $1-\gamma$. With probability γ the project continues with updated beliefs μ_{t+1}. The Bellman Equation for this problem is then given by

$$\begin{aligned} W(\mu_t) = \max_{\lambda_t\in[0,1]} \Big\{ &[\lambda_t\mu_t + [1-(1-\lambda_t)^2](1-\mu_t)]l + \\ &[(1-\lambda_t)\mu_t + (1-\lambda_t)^2(1-\mu_t)]\,[(1-\gamma)\mu_{t+1} + \gamma W(\mu_{t+1})]\Big\} \end{aligned} \tag{4}$$

where μ_{t+1} is obtained from (3).

Proposition 2. *(Efficient Liquidation) There exists a $\mu^* \in (0,1)$ such that an investor with initial belief $\mu > \mu^*$ never withdraws short term funding. An investor with initial belief $\mu < \mu^*$ exits with probability $\lambda(\mu) \in (0,1)$ in the first period. If no liquidation occurs in the first period the project is continued until maturity. We have*

$$\mu^* = \frac{2l}{1+l} > l \quad and \quad \lambda(\mu) = \frac{1}{2}\left[1 - \frac{\mu - l}{(1-\mu)l}\right]$$

Proof. see Appendix □

To gain intuition for this result, consider an active investor with belief μ. Suppose the symmetric roll over policy prescribes continuation with probability $1-\lambda$ in the current period. The benefits from continuation are given by $\mu(1-\lambda)(1-l)$ and derive from receiving 1 rather than the liquidation value l in the high payoff state. The opportunity cost of continuation arises from the potential loss of liquidation value l in the low payoff state which occurs if both investors continue. It is thus $(1-\mu)[1-(1-\lambda)^2]l$. The optimal continuation probability $\lambda(\mu)$ equalizes the marginal benefit and cost of continuation. For $\mu > \mu^*$ the marginal benefit of continuation exceeds the marginal cost for any $\lambda \in [0, 1]$. Thus liquidation is never optimal. Indeed, if $\mu > l$ the only reason for liquidating with positive probability is the creation of an option value of waiting for the co-investor. This option value is maximal if the co-investor learns the state with probability 1 by waiting one more period. Thus the maximal option value is $(1 - \mu)l$ as the state is bad with probability $1 - \mu$ in which case liquidation yields l rather than 0. The opportunity cost of liquidation in a given period is $\mu - l$. Thus liquidation can never be efficient if

$$\mu - l > (1 - \mu)l \quad \Rightarrow \mu > \mu^*$$

5 Optimal Maturity Structure

We now show how the efficient roll over policy derived in the previous section can be implemented as a symmetric PBE of the liquidation game by endowing investors with an appropriate mix of short term and long term debt claims to the investment project's payoff. Increasing the share of short term debt in an investor's portfolio increases his incentive to withdraw financing as the reward in case of unilateral exit goes up. The share of short term debt is thus a key design parameter in order to control an investor's roll over probability. It should be chosen is such a way that the incentives of an individual investor are aligned with the objectives of the planner. Here two conflicting aspects of the liquidation decision have to be traded off against each other. On the one hand a higher probability of withdrawal by one investor increases the option value of waiting for the other investor. The observation that the investment project is carried on for another period becomes more informative about the state of the world. On the other hand increasing the probability of liquidation lowers the payoff in the successful state 1. This cost is higher, the higher is the probability of success μ and lower, the higher is the liquidation value l. Proposition 3 characterises the optimal share of short term debt d^s.

Proposition 3. *For any initial belief $\mu \in (0, 1)$ there exists a face value d^s such that the equilibrium strategy $\boldsymbol{\sigma}$ of the liquidation game coincides with the efficient liquidation policy. We have*

$$d^s = \begin{cases} (1-l)\mu & \text{if } \mu < \mu^* \\ 0 & \text{if } \mu \geq \mu^* \end{cases}$$

where $\mu^* = 2l/(1+l)$.

Proof. Recall that from (1) we have $d^s = 2(\phi - l)$. ϕ needs to be such that for initial beliefs $\mu > \mu^*$ players never exit the project and for beliefs $\mu < \mu^*$ players exit with probability $\sigma_1 = \lambda(\mu) \in (0,1)$ in the first period and continue with probability 1 if no liquidation has occurred in the first period.
First consider $\mu > \mu^*$. Here continuation until maturity is the unique symmetric equilibrium outcome if $\mu > \phi$ or equivalently if $d^s < 2(\mu - l)$. If we set $d^s = 0$ this inequality is satisfied for all $\mu > \mu^*$.
Next consider $\mu < \mu^*$. In this case the efficient liquidation policy prescribes exit with probability $\lambda(\mu)$ in the first period and continuation until maturity in the absence of immediate liquidation. For this to be consistent with equilibrium behaviour in a symmetric PBE we need $0 < \phi - \mu < (1-\mu)l$ and $\sigma_1 = \lambda(\mu)$. The latter equality implies $\phi = l + (1/2)(1-l)\mu$ or equivalently $d^s = (1-l)\mu$ and it is easily verified that for this choice of ϕ the former inequalities hold for all $\mu \in (0, \mu^*)$. □

We note that for sufficiently optimistic beliefs (i.e. imprecise private information) the benefits of creating option value are outweighed by the expected costs of liquidating a successful project and thus the optimal debt structure does not involve any short term debt. The sole purpose of short term debt in this model is to create a preemption motive, which for $\mu > \mu^*$ is not desirable. If however beliefs fall below the critical value μ^* and creating option values through positive withdrawal probabilities by active investors becomes optimal, the more optimistic investors are, the higher has to be the optimal share of short term debt in their portfolio. More optimistic investors need stronger incentives to be induced to withdraw short term funding.

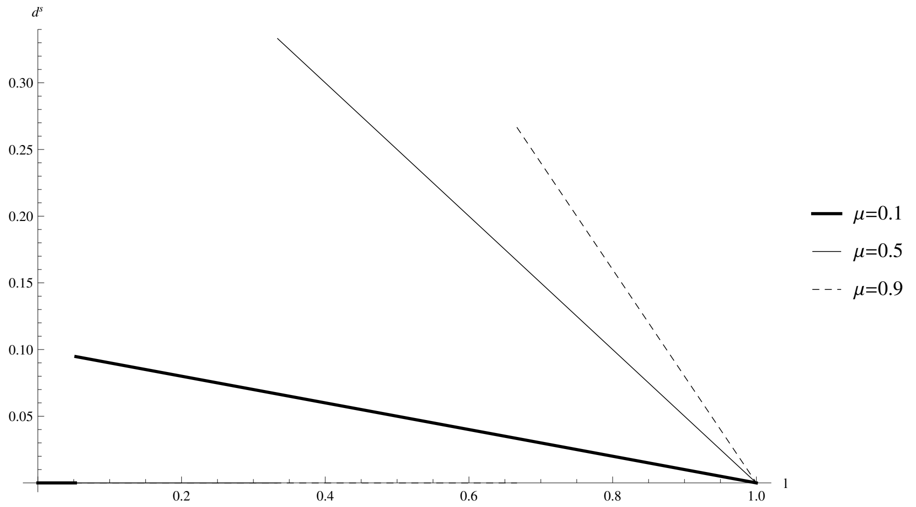

Figure 1 ***Share of short term debt and liquidation value***

Figure 1 plots optimal short term debt shares d^s against liquidation values l for different levels of beliefs. Firstly, it can be seen that the more optimistic investors are about the success of the project, the higher has to be the liquidation value in order to justify the use of short term debt as a means to incentivise information revelation. Secondly, the optimal share of short term debt becomes more sensitive to variations in the liquidation value the more optimistic investors are. Lastly, we note that the optimal maturity structure is not necessarily monotonous in investors' beliefs. For a given liquidation value l the optimal short term debt share can first increase in the level of investors' beliefs about success, but then drop off to zero as liquidation becomes too costly. [3]

6 Discussion

Communication between Investors We have ruled out direct communication between investors. If we allowed investors to pool their information, they would be able to learn the state of the world with certainty. Firstly, ruling out this possibility is motivated by desire to analyse a market where investors are anonymous and the absence of liquidation of investment projects is the only way through which privately held information can be communicated. Secondly, suppose we allowed investors to directly exchange information. Would investors have an incentive to reveal their private information truth-

[3] In figure 1 such a situation could for example occur for $l = 0.5$ where $d^s = 0$ for $\mu = 0.9$ but the optimal short term debt share is higher for $\mu = 0.5$ than for $\mu = 0.1$.

fully? Not necessarily. Assume that short term debt face values are such that $\phi > l$ and consider two investors, A and B. Suppose B reveals his information truthfully and agreed behaviour is such that in state 0 both investors exit while in state 1 both roll over. Now A does have an incentive to convince B that he has not received any bad news about the project. In this case B will roll over and A will exit receiving ϕ rather than l, which is what he would get if he revealed his information truthfully. Obviously, if preemption motives are absent, that is if $\phi = l$, truthful revelation of information becomes unproblematic.

Asymmetric Strategies In our analysis we have focused on equilibria in symmetric strategies. The equilibrium analysed above was found to be unique within this class of equilibria. But what about perfect Bayesian equilibria in asymmetric strategies? There are indeed other equilibria of the game when we allow for asymmetries in the players' behaviour. Consider for example a setting where the active players' priors satisfy the condition

$$\mu < \phi < \mu + (1-\mu)l.$$

Label players as investor A and investor B. Now consider the following equilibrium strategies: investor B, when active, always rolls over. Investor A, when active, always exits. As investor B always continues, investor A cannot learn anything from waiting. Thus as long as his payoff from exiting, which is ϕ given that B rolls over in period 1, is higher than his payoff from continuing until maturity, which is μ, he prefers to exit in period 1. Now consider investor B: if he waits in period 1, he knows the state of the project in period 2. Investor A's behaviour perfectly reveals this state. B's payoff from rolling over in period 1 is therefore $\mu + (1-\mu)(2l - \phi)$, his payoff from exiting is $\mu\phi + (1-\mu)l$. The former exceeds the latter if $\phi < \mu + (1-\mu)l$. This proves that the above strategies for A and B, which depend on the identity of the investor, indeed constitute a perfect Bayesian equilibrium of the game.
So why the focus on symmetric strategies ? We want to think of our model as applying to a market with anonymous investors without the ability to communicate other than through their investment decisions. Within such a setting it is difficult to perceive how investors would be able to coordinate on the use of asymmetric strategies.

7 Conclusion

Financing investment projects with claims whose maturity does not match the maturity of the project itself bears the risk of premature liquidation through withdrawal of funding. This risk is particularly severe if the promised payoff of the claims is not state contingent, as such payoff structures create strong preemption motives for individual investors in the event of bad news about the project's outlook.

Here we have shown that such roll over risk, which in our model arises from a maturity mismatch due to short term debt financing of a long term project with risky payoffs, can improve investment decisions by making investors' actions more informative. We have seen that the ability to learn about the risky payoffs of the project by observing other investors' funding decisions creates an option value for waiting in order to gather additional information. This informational externality induces investors to roll over their debt rather than to act on private information about payoff. The informational content of funding decisions is reduced. Rewarding the early withdrawal of funding through preferential allocation of liquidation revenue counteracts this force. This is what short term debt with its fixed face value achieves in our model. However if these preemption motives are too strong, investors exit immediately upon receiving bad news and nothing can be learned from other investors' funding decisions. We have derived the optimal share of short term debt to counterbalance such informational and payoff externalities. We have seen that if the private information received by investors is not sufficiently precise, the optimal share of short term debt financing is zero. The opportunity cost of liquidating a profitable project outweighs the option value created by the ability to observe funding decisions based on private information. For sufficiently precise private information about the project's payoff, the optimal share of short term debt is strictly positive. The preemption motive it creates for investors facilitates information revelation and enables the efficient liquidation of unprofitable investments. Optimal short term debt shares are found to decrease in the precision of the private information received by investors and the liquidation value of the project. Better informed investors need weaker incentives to act on their private information. Equally, incentives to withdraw funding prematurely can be weakened if the opportunity costs of liquidation are low.

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7.1 Proofs

Proof of Proposition 1

Proof. We need to prove that liquidation only occurs in the first period and only does so with strictly positive probability for beliefs $\mu < \mu^* \equiv 2l/(1+l)$.

First consider a simplified setting in which liquidation can only occur in the first period. If the project is not liquidated by either party in the first period it will continue until maturity. In this case the planner's objective function is

$$\left\{\mu\lambda + (1-\mu)\left[1-(1-\lambda)^2\right]\right\} l + \left[\mu(1-\lambda) + (1-\mu)(1-\lambda)^2\right]\mu'$$

where λ is the symmetric liquidation probability. By Bayes' Rule we have

$$\mu' = \frac{\mu}{\mu + (1-\mu)(1-\lambda)}$$

The objective function thus simplifies to

$$\lambda\left[(2-\lambda)(1-\mu) + \mu\right] l + (1-\lambda)\mu$$

which is strictly concave in λ. Here the optimal liquidation probability, which we designate by $\lambda(\mu)$, is

$$\lambda(\mu) = \begin{cases} \frac{1}{2}\left[1 - \frac{\mu - l}{(1-\mu)l}\right] & \text{if } \mu < \mu^* \\ 0 & \text{otherwise.} \end{cases}$$

Now consider the Bellman Equation of the original planner problem (4). To simplify the analysis we subtract μ from both sides and define $\tilde{W}(\mu) = W(\mu) - \mu$. Together with $\mu = [1 - \lambda(1-\mu)]\mu'$ which follows from Bayes' Rule (3) we obtain

$$\tilde{W}(\mu) = \max_{\lambda\in[0,1]} \left\{\lambda\left[(2-\lambda)(1-\mu)l - (1-l)\mu\right] + (1-\lambda)(1-\lambda[1-\mu])\gamma\tilde{W}(\mu')\right\}$$

Suppose the policy of liquidating with probability $\lambda(\mu)$ in the initial period upon receiving a bad signal and never thereafter is also the optimal policy for the original problem. Then the value function corresponding to this policy, namely

$$G(\mu) = \begin{cases} \lambda(\mu)\left[(2-\lambda(\mu))(1-\mu)l - (1-l)\mu\right] & \text{if } \mu < \mu^* \\ 0 & \text{otherwise} \end{cases}$$

should also satisfy the above Bellman Equation.
To see that this is the case we substitute G into the Bellman Equation and verify that it indeed solves the functional equation, that is

$$G(\mu) = \max_{\lambda\in[0,1]} \left\{ \lambda\left[(2-\lambda)(1-\mu)l - (1-l)\mu\right] + (1-\lambda)(1-\lambda[1-\mu])\gamma\, G\left(\frac{\mu}{1-\lambda(1-\mu)}\right)\right\}$$

The function inside the curly brackets is strictly concave in λ with first derivative given by

$$\begin{cases} (1-\gamma)[(1-\mu)l-(\mu-l)-2\lambda(1-\mu)l] & \text{for } \lambda < \lambda(\mu) \\ (1-\mu)l-(\mu-l)-2\lambda(1-\mu)l & \text{for } \lambda \geq \lambda(\mu) \end{cases}$$

and second derivative

$$\begin{cases} -2(1-\gamma)(1-\mu)l < 0 & \text{if } \lambda < \lambda(\mu) \\ -2(1-\mu)l < 0 & \text{if } \lambda \geq \lambda(\mu) \end{cases}$$

For $\mu < \mu^*$ the maximum is achieved by an interior solution characterized by the first order condition

$$(1-\mu)l-(\mu-l)-2\lambda(1-\mu)l = 0 \;\;\Leftrightarrow\;\; \lambda(\mu) = \frac{1}{2}\left[1-\frac{\mu-l}{(1-\mu)l}\right]$$

For $\mu \geq \mu^*$ the maximum is achieved by the corner solution $\lambda(\mu) = 0$. Hence G indeed satisfies the Bellman Equation. □